Q1. (a) 6 prisoners are to be put in 6 cells, one to a cell. How many ways can they be arranged?

(b) In Subuloland, number plates have a letter from A to J flowed by a number from 1 to 9, then a letter from T to Z. How many different number plates are possible?

(c) Students are grades A, B, C, D or E on a test. How many students must you ask to be sure of getting two with the same grade?

Q2. (a) A pitcher, a batter and a catcher have to be chosen from a team of 6 people. In how many ways can this be done?

(b) A committee of 4 must be chosen from 12 applicants. How many different committees are possible?

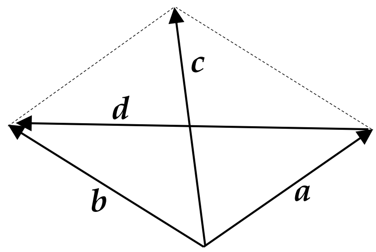
(c) A mixed netball team of 4 girls and 3 boys is to be chosen from 10 girls and 6 boys. In how many ways can this be done?

Q3. Five books are lined up on a shelf. Show that the probability that the three dictionaries are together is 30%.

Q4. Write the expansion of $(x + t)^6$. Give the coefficients as combinations.

Q5. (a) Express vectors $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

(b) If $\boldsymbol{a} = 4\boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{b} = -5\boldsymbol{i} + 6\boldsymbol{j}$, express $\boldsymbol{c} + 2\boldsymbol{d}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.



Q6. (a) Express the vector $3\boldsymbol{i} + 4\boldsymbol{j}$ in polar form, showing working.

(b) Express the vector with magnitude 4 and direction 45° in matrix Cartesian form, showing working.

Q7. (a) Find the dot product of 10 m NE and 5 m E

(b) Find the dot product of $3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{i} - 3\boldsymbol{k}$

(c) Find the angle between the vectors $2\boldsymbol{i} + 6\boldsymbol{j}$ and $2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$

Q8. A robot moves such that its position at any time t is given by

$$\boldsymbol{p} = (t + 2)\boldsymbol{i} + (t^2 - t)\boldsymbol{j}$$

Find the equation of the path of the robot in terms of x and y.

Q9. (a) Find the matrix for a rotation of –90° on the 2-d Cartesian plane.

(b) Find the co-ordinates of the point (3, 5) after the transformation $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$.

Q1. Find n if ${}^{n}P_7 = 72 \times {}^{n-2}P_5$

Q2. 10 men lived in 10 tents in a paddock. One night they all went out socialising together. When they came home they weren't thinking as well as they normally do and they couldn't work out which tent was theirs. Show that the probability that at least 8 of the men slept in their own tent that night is approximately 0.00127%.

What assumption must you make to get this result and how would not making the assumption affect the result?

Q3. Pedro took an ice cube out of his drink, tied a piece of cotton to it and sat it on a board sloping at 15°. Assume there is no friction between the ice and the board. He held the ice cube in place by holding the other end of the cotton. The cotton made an angle of 45° with the horizontal, sloping in the same direction as the board. If the tension in the cotton was 50 N, find the mass of the ice cube. Assume that g = 9.8 m/s^2.

Comment on the reasonableness of your answer.

Q4. P is transformed to P' using $\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$, then P' is transformed to P'' using $\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$.

If the co-ordinates of P'' are (4, –6), what were the co-ordinates of P?

Generalise this solution to find the coordinates of P if transforming it using matrix $\boldsymbol{M}_1$ followed by $\boldsymbol{M}_2$ moves it to the point P'' at (x, y).

Q5. Find a single matrix that produces the result of the following sequence of transformations:

- a dilation of 2 about the x-axis, then
- a dilation of 2 about the y-axis, then
- a rotation of 90° about the origin, then
- a rotation of 130° about the origin, then
- a rotation of –40° about the origin, then
- a reflection in the x-axis, and finally
- a reflection in the line $y = -x$.

Note: You have learnt how to find a matrix to represent multiple transformations by multiplying the matrices corresponding to the individual transformations. It is possible to use this method here, but it will be very tedious and difficult. There is a much easier way. Try to find it.

What are the strengths and limitations of the method you used?

Year 11 Maths C Term 3 KAPS Solutions

Q1. a) 5 x 4 x 10 = 200 different outfits

b) $^5P_5$ = 120 ways

c) $n + 1 = 7 + 1$

$= 8\ students$

Q2. a) $^5P_3$ = 60 ways

b) $^{10}C_4$ = 210 ways

c) $^{10}C_5 \times {}^8C_3$

$= 252 \times 56$

$= 14112$

Q3. $5!\ arrangements\ of\ books$

$3!\ arrangements\ of\ dictionaries$

$3!\ arrangements\ of\ books\ when\ dictionaries\ are\ treated\ as\ one\ entity$

$P = \frac{3! \times 3!}{5!}$

$= 0.3$

$\therefore 30\%$

Q4. Seventh row of Pascals triangle

1 7 21 35 35 21 7 1

$(x + a)^7 = x^7 + 7ax^6 + 21a^2x^5 + 35a^3x^4 + 35a^4x^3 + 21a^5x^2 + 7a^6x + a^7$

Q5. a) $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$

$\boldsymbol{d} = \boldsymbol{b} - \boldsymbol{a}$

b) $\boldsymbol{c} = (4\boldsymbol{i} + 3\boldsymbol{j}) + (-5\boldsymbol{i} + 4\boldsymbol{j})$

$= -\boldsymbol{i} + 7\boldsymbol{j}$

$2\boldsymbol{d} = 2(\boldsymbol{b} - \boldsymbol{a})$

$= 2\left(-5\boldsymbol{i} + 4\boldsymbol{j} - (4\boldsymbol{i} + 3\boldsymbol{j})\right)$

$= 2(-9\boldsymbol{i} + \boldsymbol{j})$

$= -18\boldsymbol{i} + 2\boldsymbol{j}$

$\boldsymbol{c} + \boldsymbol{2d} = -37\boldsymbol{i} + 11\boldsymbol{j}$

Q6. a) $|\boldsymbol{u}| = \sqrt{3^2 + 4^2}$

$|\boldsymbol{u}| = 5$

$\theta = tan^{-1}\left(\frac{4}{3}\right)$

$= 53.13°$

b) $|\boldsymbol{u}| = 4$

$\theta = 45°$

$x = |\boldsymbol{u}| \cos\theta$

$= 4\cos 45$

$= 2\sqrt{2}$

$y = |\boldsymbol{u}| \sin\theta$

$= 4\sin 45$

$= 2\sqrt{2}$

$\therefore\ 2\sqrt{2}\,\boldsymbol{i} + 2\sqrt{2}\,\boldsymbol{j}$

Q7. a) $\boldsymbol{u}.\boldsymbol{v} = 10 \times 20\cos 45$

$= 100\sqrt{2}$

b) $\boldsymbol{u}.\boldsymbol{v} = 3 \times 1 + 2 \times 0 + 1 \times -3$

$= 0$

c) $\theta = cos^{-1}\left(\frac{x_1x_2+y_1y_2+z_1z_2}{|\boldsymbol{u}||\boldsymbol{v}|}\right)$

$= cos^{-1}\left(\frac{-2}{\sqrt{40}\sqrt{14}}\right)$

$= 94.85°$

Q8. $x = 2t \quad and\ y = t^2 + 6t$

$t = \frac{x}{2}$

$sub\ t = \frac{x}{2}\ into\ y = t^2 + 6t$

$y = \left(\frac{x}{2}\right)^2 + 6\frac{x}{2}$

$y = \frac{x^2}{4} + 3x\ is\ the\ equation\ of\ the\ path\ of\ the\ particle$

Q9. (a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$

Year 11 Maths C Term 3 MAPS Solutions

Q1.

$${}^{n}P_{7} = 56 \times {}^{n-2}P_{5}$$

$$\frac{n!}{(n-7)!} = 56 \times \frac{(n-2)!}{(n-2-5)!}$$

$$\frac{n!}{(n-7)!} = 56 \times \frac{(n-2)!}{(n-7)!}$$

$$n! = 56(n-2)!$$

$$n \times (n-1) \cancel{\times (n-2) \times \ldots \times 3 \times 2} \times 1 = 56 \cancel{\times (n-2) \times \ldots \times 3 \times} 2 \times 1$$

$$n(n-1) = 56$$

$$n^2 - n - 56 = 0$$

$$n = 8 \text{ or } \cancel{-7}$$

$$\therefore n = 8$$

Q2.

There are 10! ways that the books could be handed back.

Number of arrangements with all correct is 1

Number of arrangements with 9 correct and 1 wrong is 0 because 9 cannot be correct and the 10th one wrong.

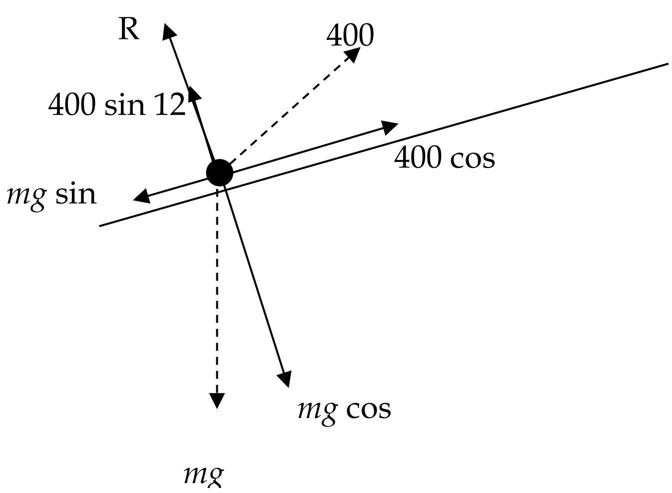
Number of arrangements with 8 correct and 2 wrong is the number of arrangements with all are correct but two are swapped over. This can be done in 10C2 ways, i.e. 45 ways.

Total number of allowable arrangements = 46.

So the probability is 46 ÷ 10!, which is about 0.000 0127, i.e. 0.00127%.
To get this result, it must be assumed that the books were distributed at random. If the teacher makes any attempt to give students their own books, the probability will increase. Any attempt to give them the wrong books will reduce the probability.

Q3.

Resolving forces parallel and perpendicular to the slope



Considering forces parallel to the slope, $mg \sin 15° = 400 \cos 12°$

$$mg = 1512$$

$$m = 154 \text{ kg}$$

So the mass of the cart and passengers is 154 kg.

This seems a bit light for a cart and 3 passengers, but then 400 N seems a bit small. So it is probably correct given the information in the question.

Q4.

$\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} P = \begin{pmatrix} 8 \\ -12 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 \\ -1 & -4 \end{pmatrix} P = \begin{pmatrix} 8 \\ -12 \end{pmatrix}$

$P = \begin{pmatrix} 2 & 0 \\ -1 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ -12 \end{pmatrix}$

$P = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ using a calculator.

So P was at (4, 2)

If P is transformed using M1 then M2 to (x, y), then $\begin{pmatrix} x \\ y \end{pmatrix} = M_2 M_1 P$

So $P = (M_2 M_1)^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$

Q5.

Following the point (1, 0) through the seven transformations in sequence, we find it arrives at (0, –2).

Following the point (0, 1) through the seven transformations in sequence, we find it arrives at (2, 0).

Hence the single matrix is $\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$.

The strength of this method is that it is very quick and easy. There are no limitations.