TASK A – Shuffling

Q1.

Take 10 cards, the A-10 of a suit. Shuffle the cards.

a) How many different orders could the cards end up in?

b) How many different hands of 4 cards is it possible to deal from the pack of 10? (Note: A, 4, 7, 2 is the same hand as 7, A, 4, 2.)

c) If you shuffled the 10 cards and dealt someone 5 of them, what is the probability that their hand would include the A, 2 and 3?

Q2.

Riffling is a shuffling technique in which you split the pack into two equal stacks, take one stack in each hand, then interleave the two stacks. Few people can do a perfect riffle, but you can simulate one by counting out the top half of the pack, then taking cards alternately from the top and bottom halves to form the shuffled pack.

Try this with your 10 cards. Start with the cards in order, ace at the top. After riffling, they should end up (from top to bottom): A, 6, 2, 7, 3, 8, 4, 9, 5, 10.

a) If you riffle a pack repeatedly, the cards will eventually come back to the original order. Explain why this must be so. Your explanation must be valid for any even number of cards.

b) How many riffles will it take for 10 cards to come back to the original order?

c) Find how many riffles for 2, 4, 6 and 8 cards, then for 52.

Perform the riffles on paper or with a computer simulation rather than with the actual cards and hand in the working with your answer. Hand-written working is fine.

From 2011

Try this with your 10 cards. Start with the cards in order, ace at the top. After riffling, they should end up (from top to bottom): A, 6, 2, 7, 3, 8, 4, 9, 5, 10.

Q2. If you riffle a pack repeatedly, the cards will eventually come back to the original order. Explain why this must be so. Your explanation must be valid for any even number of cards.

Q3. How many riffles will it take for 10 cards to come back to the original order?
Find how many riffles for 2, 4, 6 and 8 cards, then for 52. Perform the riffles on paper or with a computer simulation rather than with the actual cards and hand in the working with your answer. Hand-written working is fine.

Q4. Using your working from Q3, find a formula that will allow you to work out how many riffles it would take for any even number of cards without actually doing the riffling (or simulating it). HINT: You will need to follow where the cards go from riffle to riffle. Pay particular attention to the card that starts second from the top.
You must explain how you got from the working of Q3 to your formula. State any assumptions you make along the way.

Q5. Use your formula to find how many riffles it would take for a pack of 26 cards to return to the original order. Show working.

TASK B – Poker

Q3.

Tong is a variation on poker, in which players are dealt 6 cards instead of 5. The hand categories are similar to those for 5-card poker, but with some differences. They are:

- Ace high
- Pair
- Three pairs
- Crowded house (2 sets of 3-of-a-kind)
- Straight
- Half flush (3 cards the same suit)

For each hand category, calculate the probability of being dealt it. Thus rank the categories from least likely (and therefore top-ranked) to most likely (and therefore bottom-ranked). Consider a hand to contain a category even if it also contains a higher-ranked category. So, for example, a hand with a crowded house also contains a pair.

Q4.

Invent another variation on poker by removing some of the cards from the pack and changing the number of cards dealt to each player. Use six hand categories, at least two of which should be significantly different from those in tong or poker. Your game needs to be original enough that it is unlikely to be thought of by other people. Also, it needs to be sufficiently involved that the calculations are not trivial.

Give a brief explanation of the game, give the hand categories with explanations where appropriate, and calculate the probabilities associated with each category.

Run a simulation of your game by using the Excel macro provided on scholaris, altering the appropriate parts as necessary. Simulate the dealing of 100 hands and find the relative frequency of each hand category.

a) Present your information in a table showing, for each category, the calculated probability and the relative frequency obtained from the simulation.

b) Email a copy of your Excel file with all the data for the 100 simulations in it to Mr Ilsley.

c) Comment on the reasonableness of your results and whether the simulations support your calculations.

Year 11 Maths C Semester 2 Assignment 2013 - Solutions

1. a) $10! = 3{,}628{,}800$

   b) ${}^{10}C_4 = 210$

   c) Probability = ${}^{7}C_5 / {}^{10}C_5 = \frac{1}{12}$

2. a) Proof

   b,c) Need to show riffling

No. of Cards	No. of riffles taken
10	6
2	1
4	2
6	4
8	3
52	8

4.

Explanation must include:

- top and bottom cards never move
  - The order of the cards
  - The position of cards as they move and in terms of position number in powers of 2
  - Position given by 2^n with n the number of riffles
  - Once reaches end of deck and cycles back through the formula changes to $p \equiv 2^n (mod\ c - 1) with\ c\ the\ number\ of\ cards$
  - To be in the original order need p=1

5.

Solve $2^n \equiv 1 (mod\ c - 1)$ for n

n=20

Need to show steps to get the solution

3. Six-Card Poker

Number of possible combinations $\binom{52}{6} = 20{,}358{,}520$ possible combinations

Hand	Combinations	Probability
Ace high	$^{52}C_6 - {}^{48}C_6$ 1-48/52*47/51*46/50*45/49*44/48*43/47	0.397230 0.397230
Pair	1 – 1*48/51*44/50*40/49*36/48*32/47	0.655
Three Pair	$^{13}C_3*(^4C_2)\text{^}3 + {}^{13}C_1*{}^4C_4*{}^{12}C_1*{}^4C_2$ 3 different pairs . . four + pair	0.003034405251 0.003080381089
Crowded house (2 sets of 3-of-a-kind)	$\binom{13}{2}\binom{4}{3}^2 = 1{,}248$	0.0000613011
Straight (of 6 cards)	$9*(^4C_1)\text{^}6 = 36{,}864$	0.001810740663
Half flush (3 cards the same suit)	$(^{13}C_3*{}^{39}C_3+{}^{13}C_4*{}^{39}C_2+{}^{13}C_5*{}^{39}C_1+{}^{13}C_6)*4$ $(^{13}C_3*(^{39}C_3 - {}^{13}C_3*3)+{}^{13}C_4*{}^{39}C_2+{}^{13}C_5*{}^{39}C_1+{}^{13}C_6)*4$	0.62784* 0.5796

* The first answer will be accepted. The second is correct.
Solution is to include working for their calculations.

4. Response to include:
   - Explanation of the game including ranks, suits etc.
   - Table showing for each category, the calculated probability and the relative frequency obtained from the simulation
   - the data produced by the simulation in an emailed spreadsheet
   - Comment on the reasonableness of the results and whether the simulations support the calculations

Year 11 Maths C Semester 2 Assignment Solutions - Mitch

5. a) $10! = 3{,}628{,}000$

   b) ${}^{10}C_4 = 210$

   c) Probability = ${}^{7}C_2 / {}^{10}C_5 = \frac{1}{12}$

   d) Need to show riffling

No. of Cards	No. of riffles taken
2	1
4	2
6	4
8	3
10	6
52	8

6. Explanation
7. Six-Card Poker

Number of possible combinations $\binom{52}{6} = 20{,}358{,}520$ possible combinations

Hand	Example	Combinations	Probability
High Card		6,985,044	0.3431017579
One Pair		$\binom{13}{1}\binom{4}{2}\binom{12}{4} \times 4^4 = 9{,}884{,}160$	0.4855048402
Two Pair		$\binom{13}{2}\binom{4}{2}^2\binom{11}{2} \times 4^2 = 2{,}471{,}040$	0.1213762101
Three of a Kind		$\binom{13}{1}\binom{4}{3}\binom{12}{3}\binom{4}{1}^3 = 732{,}160$	0.0359633215
Three of a Kind plus a Pair		$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}\binom{11}{1} \times 4 = 164{,}736$	0.00809174733
Three Pair		$\binom{13}{3}\binom{4}{2}^3 = 61{,}776$	0.003034405251
Straight		$9 \times \binom{4}{1}^6 - 36 = 36{,}828$	0.001809168
Four of a Kind		$\binom{13}{1}\binom{4}{4}\binom{12}{2}\binom{4}{1}^2 = 13{,}728$	0.0006743122
Flush		$\binom{13}{6}\binom{4}{1} - 36 = 6{,}828$	0.000335387
Two Three of a Kind		$\binom{13}{2}\binom{4}{3}^2 = 1{,}248$	0.0000613011
Full House (Four of a kind + a pair)		$\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{2} = 936$	0.000045975
Straight Flush		$9 \times \binom{4}{1} - 4 = 32$	0.000001571
Royal Flush		$\binom{4}{1} = 4$	0.0000001964

Total		$\binom{52}{6} = 20,358,520$	**1**

Solution is to also include examples of each hand, and explanation on their calculations

8. a)

   Response to include:

   - Explanation of the game including ranks, suits etc.
   - Table similar to Q4 with calculated combinations and probabilities in ranked order

   b)

   i) Simulation of their game for 100 iterations to give relative frequency of each hand with results in a table

   ii) Comment on the reasonableness of results

   – i.e. not all hands possible – odds etc., yet randomness of excel allows a reasonable result even though 10 hands dealt does diminish the chances as 5th card dealt is dealt from reduced pack and hence may have removed cards needed for achieving some hands