

Structures and Patterns

S1. Expand:

(a) $\sum_{i=1}^5 i^2$ (b) $\sum_{r=0}^2 3^r$ (c) $\sum_{r=5}^{\infty} \frac{1}{r}$ (d) $\sum_{r=0}^{\infty} \frac{x^r}{r!}$

S2. Evaluate:

(a) $\sum_{r=1}^4 r^2$ (b) $\sum_{i=0}^2 2^i$ (c) $\sum_{a=0}^5 (2a + 5)$ (d) $\sum_{r=8}^9 (r^2 - 3r + 1)$
(e) $\sum_{r=1}^5 \frac{1}{2^r}$ (f) $\sum_{c=0}^2 5^c$ (g) $\sum_{r=0}^{\infty} 2^{-r}$ (h) $\sum_{r=0}^{20} 2^r$

S3. Simplify:

(a) $\sum_{r=0}^{2000} 2^r$ (b) $\sum_{i=2}^{40} \left(\frac{3}{4}\right)^i$ (c) $\sum_{a=0}^x 2a$ (d) $\sum_{r=1}^p (3r + 1)$
(e) $\sum_{r=1}^{100} 2^{r+3}$ (f) $\sum_{i=2}^{40} \frac{1}{2^i}$ (g) $\sum_{a=0}^x 2^{2a}$ (h) $\sum_{r=1}^p (2^r + 5)$

S4. Use the method of finite differences to find the n th term in each of the following sequences:

- (a) 6, 11, 16, 21, 26, 31, ...
(b) -2, 5, 16, 31, 50, ...
(c) -1, 13, 49, 119, 235, 409, ...
(d) 1, 5, 14, 30, 55, 91, ...

S5. Use the method of finite differences to find:

(a) $\sum_{r=1}^x r$ (b) $\sum_{r=1}^n 4r$ (c) $\sum_{r=1}^x (2 - r)$ (d) $\sum_{r=1}^n r^2$
(e) $\sum_{r=4}^x (3r + 1)$ (f) $\sum_{r=3}^x 4r^2$ (g) $\sum_{r=1}^x (r^2 - 3r)$ (h) $\sum_{r=1}^x 6$

S6. Using finite differences or otherwise, find the n th term in each of these sequences:

- (a) 1, 4, 7, 10, ...
(b) 0.125, 0.5, 2, 8, ...
(c) 2, 6, 18, 54, ...
(d) 153, 136, 119, 102, ...
(e) 2, 5, 10, 17, 26, 37, ...
(f) 2, 3, 5, 9, 17, 33, ...
(g) 6, 22, 54, 118, 246, 502, ...
(h) 2, 20, 74, 236, 722, ...
(i) 1, 3, 7, 15, 31, 63, ...

- S1. (a) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$ (b) $3^0 + 3^1 + 3^2$
(c) $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots$ (d) $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
- S2. (a) 30 (b) 7 (c) 60 (d) 96
(e) $3^{1/32}$ (f) 31 (g) 2 (h) 2 097 151
- S3. (a) $2^{2001} - 1$ (b) $\frac{9}{4} \left(1 - \left(\frac{3}{4} \right)^{39} \right)$ (c) $x^2 + x$ (d) ${}^{3/2}p(p+1) + p$
(e) $16(2^{100} - 1)$ (f) $2^{-1} - 2^{-40}$ (g) ${}^{1/3}(4^{x+1} - 1)$ (h) $2^{p+1} - 2 + 5p$
- S4. (a) $5n + 1$ (b) $2n^2 + n - 5$ (c) $2x^3 - x^2 + 3x - 5$ (d) ${}^{1/3}x^3 - {}^{1/2}x^2 + {}^{1/6}x$
- S5. (a) ${}^{1/2}x^2 + {}^{1/2}x$ (b) $2x^2 + 2x$ (c) $-{}^{1/2}x^2 + 1{}^{1/2}x$ (d) ${}^{1/3}x^3 + {}^{1/2}x^2 + {}^{1/6}x$
(e) ${}^{3/2}x^2 + {}^{5/2}x - 21$ (f) ${}^{3/2}x^3 + 2x^2 + {}^{2/3}x - 20$ (g) ${}^{1/3}x^3 - x^2 - {}^{1/3}x$ (h) $6x$
- S6. (a) $3n - 2$ (b) $0.125 \times 2^{n-1}$ (c) $2 \times 3^{n-1}$ (d) $17n + 136$
(e) $n^2 + 1$ (f) $2^{n-1} + 1$ (g) $2^{n+3} - 10$ (h) $3^{n+1} - 7$
(i) $2^n - 1$