Differential Equations

1. $\frac{dy}{dx} = \frac{x}{2y}$
2. $\frac{dy}{dx} = \frac{y}{x}$
3. $\frac{dy}{dx} = \frac{3x^2 + 3x}{y}$
4. $\frac{dy}{dx} = 2xy$
5. $\frac{dy}{dx} = \frac{x}{y^3}$
6. $\frac{dy}{dx} - x^2y^2 = 0$
7. $8y\frac{dy}{dx} = 12x(x^2 - 7)^5$
8. $\frac{y^2}{2x}\frac{dy}{dx} = \frac{(x^2 - 7)^2}{(2y^3 + 1)^5}$
9. $(2x + 1)\sin y - (x^2 + x)\cos y\frac{dy}{dx} = 0$
10. $2y\frac{dy}{dx} - 16x = 4xy^2$
11. $\frac{4}{y^2}\left(\frac{dy}{dx}\right)^2 = \cos^2 x$

Answers

1. $y = \pm\sqrt{\frac{x^2}{2} + c}$
2. $y = Ax$
3. $y = \pm\sqrt{2x^3 + 3x + c}$
4. $y = Ae^{x^2}$
5. $y = \pm\sqrt[4]{2x^2 + c}$
6. $y = \frac{-3}{x^3 + c}$
7. $y = \pm\frac{\sqrt{(x^2 - 7)^6 + c}}{2}$
8. $y = \sqrt[3]{\frac{-1 \pm \sqrt[6]{12(x^2 - 7)^3 + c}}{2}}$
9. $y = \sin^{-1} A(x^2 + x)$
10. $y = \pm\sqrt{Ae^{2x^2} - 4}$
11. $y = Ae^{\pm\frac{\sin x}{2}}$

Solutions

Differential Equations

① $\frac{dy}{dx} = \frac{x}{2y}$

$\int 2y\,dy = \int x\,dx$

$y^2 = \frac{x^2}{2} + C$

$y = \pm\sqrt{\frac{x^2}{2} + C}$

② $\frac{dy}{dx} = \frac{y}{x}$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln y = \ln x + C$

$\ln y = \ln x + \ln d$

$\ln y = \ln dx$

$y = dx$

③ $\frac{dy}{dx} = \frac{3x^2 + 3x}{y}$

$\int y\,dy = \int (3x^2 + 3x)\,dx$

$\frac{y^2}{2} = x^3 + \frac{3}{2}x^2 + C$

$y^2 = 2x^3 + 3x^2 + d$

$y = \pm\sqrt{2x^3 + 3x^2 + d}$

④ $\frac{dy}{dx} = 2xy$

$\int \frac{dy}{y} = \int 2x\,dx$

$\ln y = x^2 + C$

$y = e^{x^2 + C}$

$y = Ae^{x^2}$

⑤ $\frac{dy}{dx} = \frac{x}{y^3}$

$\int y^3\,dy = \int x\,dx$

$\frac{y^4}{4} = \frac{x^2}{2} + C$

$y^4 = 2x^2 + d$

$y = \pm\sqrt[4]{2x^2 + d}$

⑥ $\frac{dy}{dx} - x^2y^2 = 0$

$\frac{dy}{dx} = x^2y^2$

$\frac{dy}{y^2} = x^2\,dx$

$\int y^{-2}\,dy = \int x^2\,dx$

$-y^{-1} = \frac{x^3}{3} + C$

$y^{-1} = -\left(\frac{x^3}{3} + C\right)$

$y = \frac{-1}{\frac{x^3}{3} + C}$

$y = \frac{-3}{x^3 + d}$

(7) $8y\frac{dy}{dx} = 12x(x^2-7)^5$

$\int 8y\,dy = \int 12x(x^2-7)^5\,dx$

$4y^2 = (x^2-7)^6 + c$

$y^2 = \frac{(x^2-7)^6+c}{4}$

$y = \pm\frac{\sqrt{(x^2-7)^6+c}}{2}$

(8) $\frac{y^2}{2x}\frac{dy}{dx} = \frac{(x^2-7)^2}{(2y^3+1)^5}$

$\int y^2(2y^3+1)^5\,dy = \int 2x(x^2-7)^2\,dx$

$\frac{(2y^3+1)^6}{36} = \frac{(x^2-7)^3}{3} + c$

$(2y^3+1)^6 = 12(x^2-7)^3 + d$

$2y^3+1 = \pm\sqrt[6]{12(x^2-7)^3+d}$

$2y^3 = -1 \pm \sqrt[6]{12(x^2-7)^3+d}$

$y^3 = \frac{-1\pm\sqrt[6]{12(x^2-7)^3+d}}{2}$

$y = \sqrt[3]{\frac{-1\pm\sqrt[6]{12(x^2-7)^3+d}}{2}}$

(9) $(2x+1)\sin y - (x^2+x)\cos y\frac{dy}{dx} = 0$

$(2x+1)\sin y = (x^2+x)\cos y\frac{dy}{dx}$

$\int\frac{2x+1}{x^2+x}\,dx = \int\frac{\cos y}{\sin y}\,dy$

$\ln(x^2+x) = \ln(\sin y) + c$

$\ln(x^2+x) + \ln A = \ln(\sin y)$

$\ln(A(x^2+x)) = \ln(\sin y)$

$A(x^2+x) = \sin y$

$y = \sin^{-1}(A(x^2+x))$

(10) $2y\frac{dy}{dx} - 16x = 4xy^2$

$2y\frac{dy}{dx} = 4xy^2 + 16x$

$2y\frac{dy}{dx} = (4y^2+16)x$

$\int\frac{2y}{4y^2+16}\,dy = \int x\,dx$

$\frac{1}{4}\ln(4y^2+16) = \frac{x^2}{2} + C$

$\ln(4y^2+16) = 2x^2 + d$

$4y^2 + 16 = e^{2x^2+d}$

$4y^2 = Ae^{2x^2} - 16$

$y^2 = Be^{2x^2} - 4$

$y = \pm\sqrt{Be^{2x^2}-4}$

(11) $\frac{4}{y^2}\left(\frac{dy}{dx}\right)^2 = \cos^2 x$

$\frac{2}{y}\frac{dy}{dx} = \pm\cos x$

$\int\frac{2}{y}\,dy = \pm\int\cos x\,dx$

$2\ln y = \pm\sin x + C$

$\ln y = \pm\frac{\sin x}{2} + d$

$y = e^{\pm\frac{\sin x}{2} + d}$

$y = Ae^{\pm\frac{\sin x}{2}}$