1. (a) Express $-\sqrt{2} + \sqrt{2}\, i$ in exponential form.

   (b) Express $\sin i$ in Cartesian form.

2. Find $\int_0^1 x\, e^{(4x^2-5)} dx$

3. Find $\int \tan x dx$

4. If $\frac{dy}{dx} = x^2 \sin x$ and $y = 3$ when $x = 1$, use integration by parts to find y.

5. Use Euler's formula to find $\int e^{2x} \sin 2x dx$.

6. Use Simpson's rule with 4 strips to evaluate $\int_1^2 \frac{x}{1+x^2} dx$.

   Is this method accurate?

7. Use complex number techniques to prove that $\cos 2x = \cos^2 x - \sin^2 x$

8. The curve $y = x^2$ between $x = 0$ and $x = 3$ is rotated about the x axis. Find the volume generated.

9. A secret has started to spread through a school of 1500 students. It is predicted that the rumour will spread according to the equation $\frac{dN}{dt} = 0.0002N(1500 - N)$, where N is the number of people who have heard the secret and t is the time in hours. At 9 a.m. 12 people had heard the secret. Solve the equation using partial fraction techniques and calculus. Hence find the number of people predicted to have heard the secret by 3:30 p.m.

   List factors that could cause the actual number to differ from that predicted by the model?

10. Ground-bound aliens measured the surface area of their planet to be 45 381 square moules. This would make its diameter 120.19 moules. There is a possible slight error of x square moules in the surface area measurement. Use calculus to find an expression for the possible error in the radius.
What are the strengths and limitations of this method for finding the error in the radius?

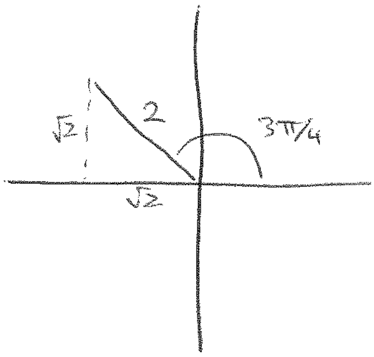
11. Find $\int_0^{\pi/2} \sin^4\theta$

12. An ornamental dish is made in the shape of part of a cone with a depression in the top. The cone is produced by rotating the line $y = 4x - 4$ around the y-axis. The depression is produced by taking the parabola $y = x^2$ and rotating it about the y-axis. The base of the dish is taken at $y = 0$, and the top is where the depression intersects the cone. The vase is made of dish with a density of 2.5 g per cubic unit. By slicing the dish into horizontal slices, find the volume of the dish and hence its mass.

13. A sine function $f(t)$ has an amplitude of 3, a mean position of 0 and a phase shift of 0. However, its period increases from $\pi/6$ when t = 0 to 4π when $t = 8\pi$. Find an algebraic formula to model the function over the domain $0 < t < 4\pi$. Feel free to experiment with your graphics calculator.

1 a) $-\sqrt{2}+\sqrt{2}i$

$= 2e^{\frac{3\pi}{4}i}$



b) $\sin\theta$

$= \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

$\sin i = \dfrac{e^{ii}-e^{-ii}}{2i}$

$= \dfrac{e^{-1}-e^{1}}{2i}$

$= \dfrac{-i}{2}\left(\frac{1}{e}-e\right)$

2 $\displaystyle\int_0^1 xe^{(4x^2-5)}\,dx$

Try

$e^{4x^2-5} \longrightarrow e^{4x^2-5} \times 8x$

$\frac{1}{8}e^{4x^2-5} \longrightarrow \cancel{\frac{1}{8}}e^{4x^2-5}\,\cancel{8}x$ ✓

$$\text{Integral} = \left[\frac{1}{8}e^{4x^2-5}\right]_0^1$$

$$= \frac{1}{8}e^{-1} - \frac{1}{8}e^{-5}$$

③ $\int \tan x \, dx$

$$= \int \frac{\sin x}{\cos x} dx$$

Try

$\ln \cos x \rightarrow \frac{1}{\cos x}(-\sin x)$

$-\ln \cos x \rightarrow -\frac{1}{\cos x}(-\sin x)$ ✓

$\therefore \int \tan x \, dx = -\ln \cos x + c$

(6) Let $y = \frac{x}{1+x^2}$

x	y
1	0·5000
1·25	0·4878
1·5	0·4615
1·75	0·4307
2	0·4000

$$\int_1^2 \frac{x}{1+x^2}\,dx = \frac{1}{12}\left(0{\cdot}5 + 4\times 0{\cdot}4878 + 2\times 0{\cdot}4615 + 4\times 0{\cdot}4307 + 0{\cdot}4\right)$$

$$= 0{\cdot}485$$

This method will be reasonably accurate, but not exact.

(7) RTP $\cos 2x = \cos^2 x - \sin^2 x$

$$\text{RHS} = \left(\frac{z+z^{-1}}{2}\right)^2 - \left(\frac{z-z^{-1}}{2i}\right)^2$$

$$= \frac{z^2+2+z^{-2}}{2^2} - \frac{z^2-2+z^{-2}}{-4}$$

$$= \frac{1}{4}\left(\cancel{2}z^2+2+z^{-2}+z^2-2+z^{-2}\right)$$

$$= \frac{1}{4}\left(2z^2+2z^{-2}\right)$$

$$= \frac{z^2+z^{-2}}{2}$$

$$= \cos 2x$$

$$= \text{LHS}$$

1 Let the surface area be A and the radius r

$$A = 4\pi r^2$$

$$\frac{dA}{dr} = 8\pi r$$

$$\frac{\Delta A}{\Delta r} \approx 8\pi r \text{ for small } r$$

$$\Delta r \approx \frac{\Delta A}{8\pi r}$$

$\Delta A = x \qquad r = 60.095$

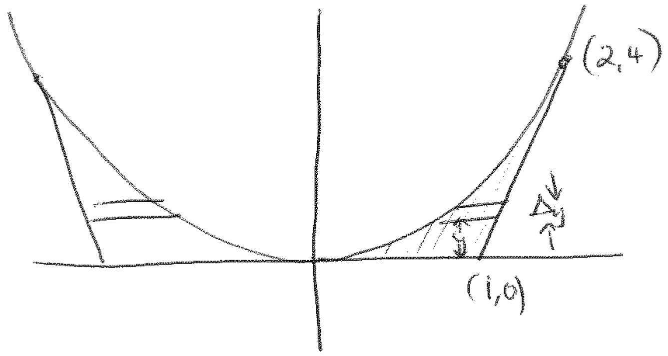
$$\therefore \Delta r = \frac{x}{1510}$$

Error in ~~diam~~ radius $= \dfrac{x}{1510}$

Error in diameter $= 2\Delta r = \dfrac{2x}{1510}$

(2) Did on board

(3)



Divide the rotated shapes into discs as shown

Rearranging the equations

$$y = x_2^2 \qquad\qquad y = 4x_1 - 4$$

$$x_2 = y^{\frac{1}{2}} \qquad\qquad 4x_1 = y + 4$$

$$x_1 = \frac{y}{4} + 1$$

Area of a disc $= \pi x_1^2 - \pi x_2^2$

$$= \pi\left(1 + \frac{y}{4}\right)^2 - \pi y$$

$$= \pi\left(1 + \frac{y}{2} + \frac{y^2}{16}\right) - \pi y$$

$$= \pi\left(1 + \frac{y}{2} + \frac{y^2}{16} - y\right)$$

$$= \pi\left(1 - \frac{y}{2} + \frac{y^2}{16}\right)$$

Sum of all discs $= \displaystyle\sum_{y=0}^{4} \pi \left(1 - \frac{y}{2} + \frac{y^2}{16}\right) \Delta y$

Exact volume $= \pi \displaystyle\int_0^4 \left(1 - \frac{y}{2} + \frac{y^2}{16}\right) dy$

$$= \pi \left[y - \frac{y^2}{4} + \frac{y^3}{48} \right]_0^4$$

$$= \pi \left[4 - \frac{4^2}{4} + \frac{4^3}{48} \right]$$

$$= \pi \left[4 - 4 + \frac{4}{3} \right]$$

$$= \frac{4\pi}{3}$$

So the volume of the dish is $\frac{4\pi}{3}$ units

And the ~~density~~ mass is $\frac{10\pi}{3}$ g

(4) I want you to puzzle over this one.