A5 – Objectives and Key Facts

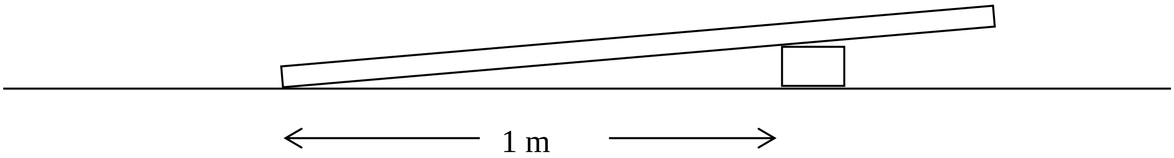
a) Understand gradient and axis intercepts
b) Convert between tables, graphs and formulae using $y = mx + c$

Algebra – Lesson 5.1

Walking the Plank

Get a plank of wood a bit over a metre long and a few bricks.

Mark out a distance of one metre on level ground. Put a brick at one end, then rest the plank on the brick as shown below.

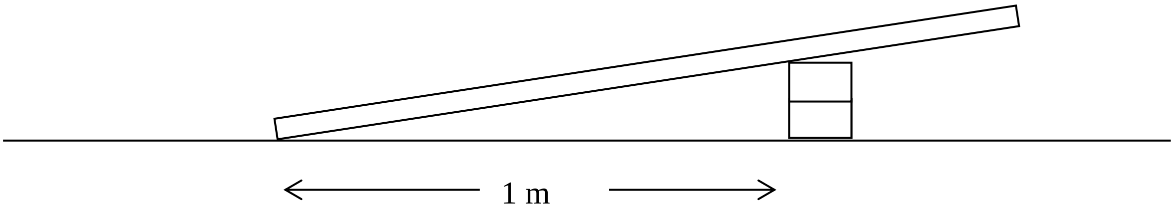


Measure the height of the brick. This will be different for different bricks, but a typical height might be 8 cm. As we have measure the horizontal distance in metres, it would be best not to mix our units, so we should convert the height to metres also. It might be 0.08 m.

This means that the plank rises 0.08 m in one horizontal metre.

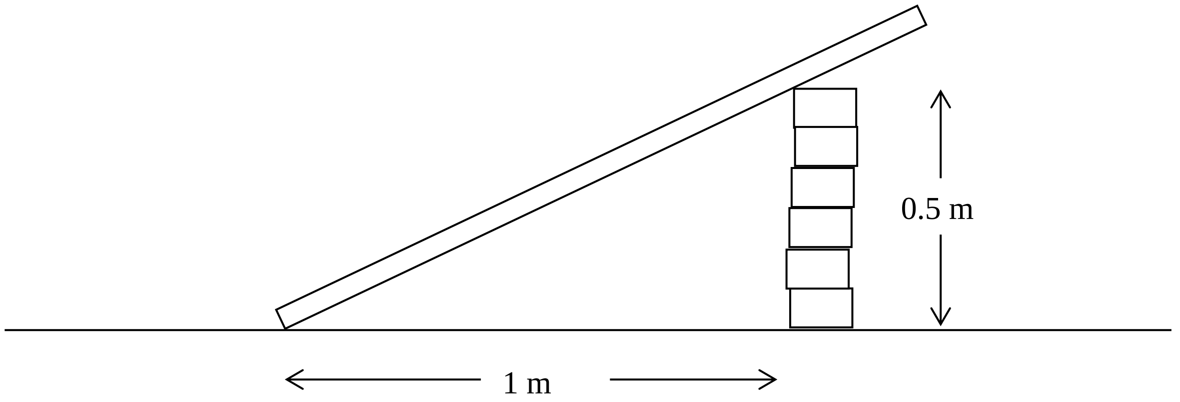
Then get someone to see if they can walk up the plank.

Then set up the plank with two bricks on top of each other to make it steeper.

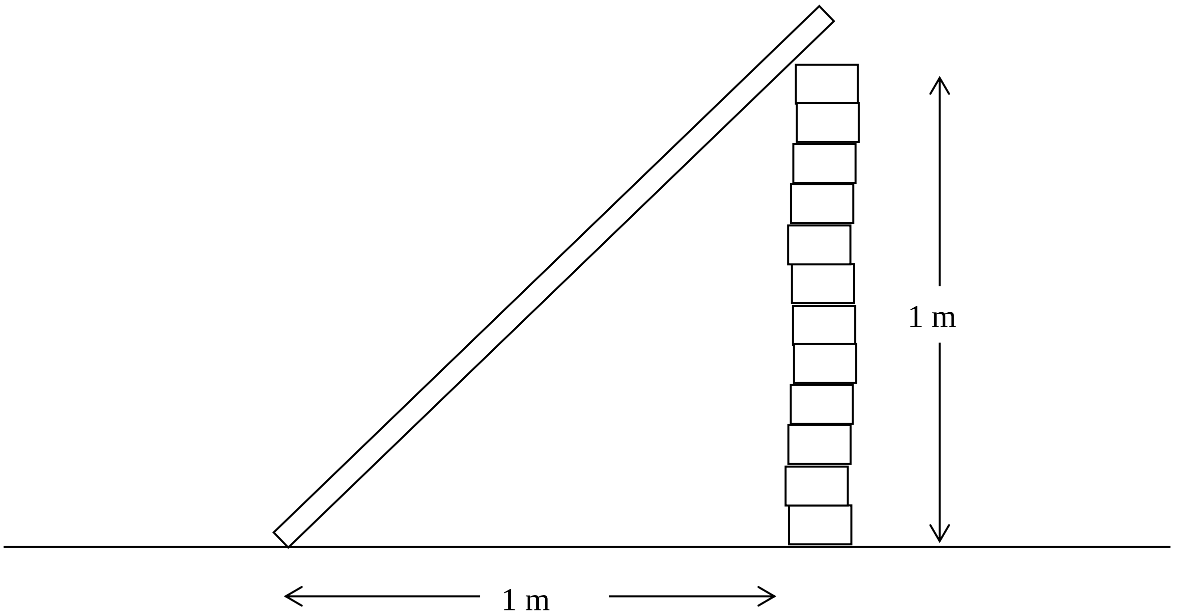


This time the plank might rise 0.16 m in one horizontal metre. Again, get someone to see if they can walk up it. Hopefully, they will be able to.

Then have a guess at the steepest we can make the plank can for people still to be able to walk up it. Make your estimate in vertical metres per horizontal metre. For instance you might guess that you could just walk up it at 0.5 vertical metres per horizontal metre:



or you might think you could manage it at 1 vertical metre per horizontal metre:



or maybe even 1.5 vertical metres per horizontal metre.

Then try it by adding bricks and trying to walk up it.

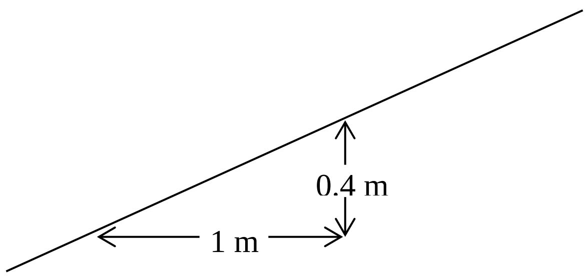
Algebra – Lesson 5.2

Building Roads

In the *Walking the Plank* activity you should have found out how steep a plank you can walk up. You would probably find that the steepest tar roads and concrete paths that you can walk up are about the same steepness as the steepest plank you could walk up. This is important to consider when people are building roads and footpaths up hills – it's no good building a footpath that no one can walk up.

One of the steepest roads around the Beenleigh area is Queens Road in Kingston. If you have ever been there, estimate how steep you think it is in vertical metres per horizontal metre.

Note that Queens Road is more than 1 m long! Hopefully you should be able to handle this. Just think about how many metres you rise as you move 1 m horizontally along the road.



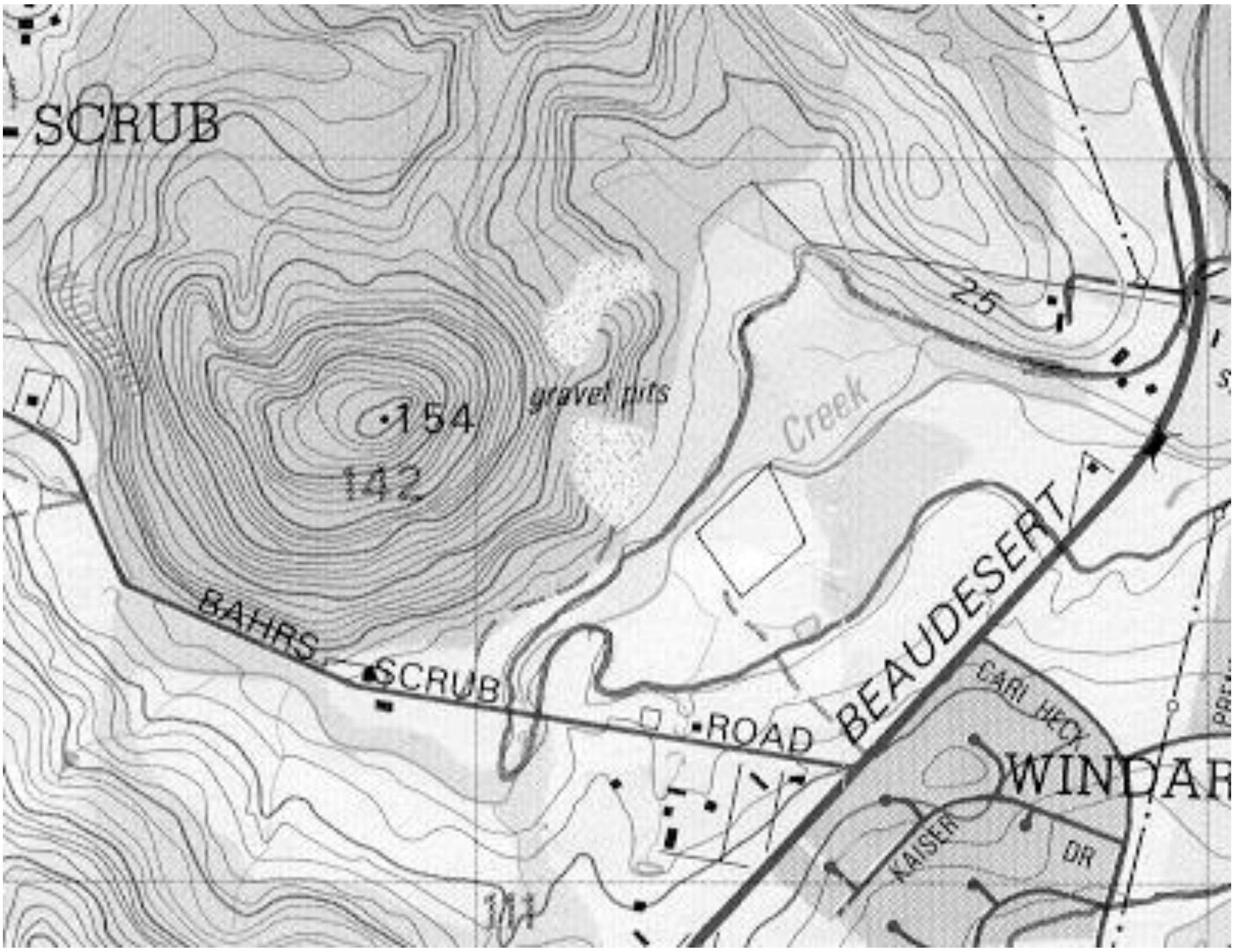
This would mean that the steepness of the road is 0.4 vertical metres per horizontal metre.

When we measured the steepness of the plank, we had the horizontal ground to make the horizontal measure along. We don't have that if we are measuring the steepness of a road. Having a spirit level available would help though. Also, if we use the diagram above, the measurements we need to take are under the ground. The council wouldn't appreciate us digging up the road. Can you think of a way around this? When you think of a way, find some sloping ground around the school and go and measure how steep it is.

If anyone in the class lives out Kingston way or is going out there, get them to measure the steepness of Queens Road.

Algebra – Lesson 5.3

The Hill behind the School



This is a topographic map of the area around Windaroo Valley State High School. It was made before the school was built, so the school doesn't show. You should be able to pick where it is though.

'Topographic' means showing the height of the land above sea level. The height is shown with contours. These are the wavy lines that cover most of the map. These contours make the hill behind the school very obvious. The top of the hill is 154 metres above sea level. That's what the dot and the 154 mean. The lines going in sort of circles around the top of the hill are contours. The smallest ring around the dot is the 150 m contour. It shows all the places where the ground is 150 m above sea level. The next one is the 145 m contour. It shows all the places where the ground is 145 m above sea level. And so on.

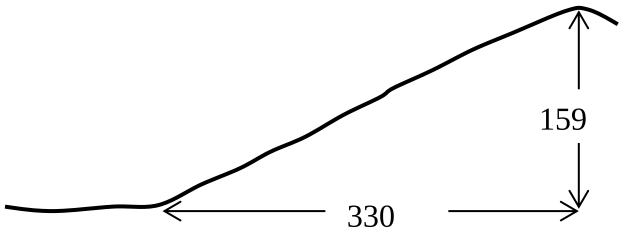
You will notice that the contours for 150 m, 125 m, 100 m, 75 m, 50 m and 25 m are darker than the others. This makes them easier to count. If you count down, you will see that the bottom of the hill is about 15 m above sea level. This is about the height of the school.

So the difference in height between the bottom and the top of the hill is about 139 m.

Now the scale of the map is 1:10 000. This means that 1 cm on the map represents 10 000 cm (that is 100 m) on the ground. Knowing this, you can measure the horizontal distance between the bottom of the hill (on the school side) and the top of the hill. Do it.

On the map it should come to about 3.3 cm. This is 3.3 lots of 100 metres in reality, ie about 330 m.

So if you walked up the hill from the school, you would walk 330 m horizontally and 159 m vertically. In profile, the hill would look something like this:



Now we know this we can work out how steep the hill is compared to the planks we walked up in the first lesson and whether it is possible to walk up it. This is how we do it.

We know that when we walk 330 m horizontally, we climb 159 m. What we need to know is how much we climb when we walk 1 m horizontally. In other words, we need to know the number of vertical metres per horizontal metre.

If we climb 159 m while going 330 m horizontally, then we will climb one 330th of the 159 m for each horizontal metre we walk. To get $\frac{1}{330}$ of 159, we divide 159 by 330. This gives us 0.48. So we walk 0.48 vertical metres per horizontal metre. In other words we can say that the steepness of the hill is 0.48 vertical metres per horizontal metre.

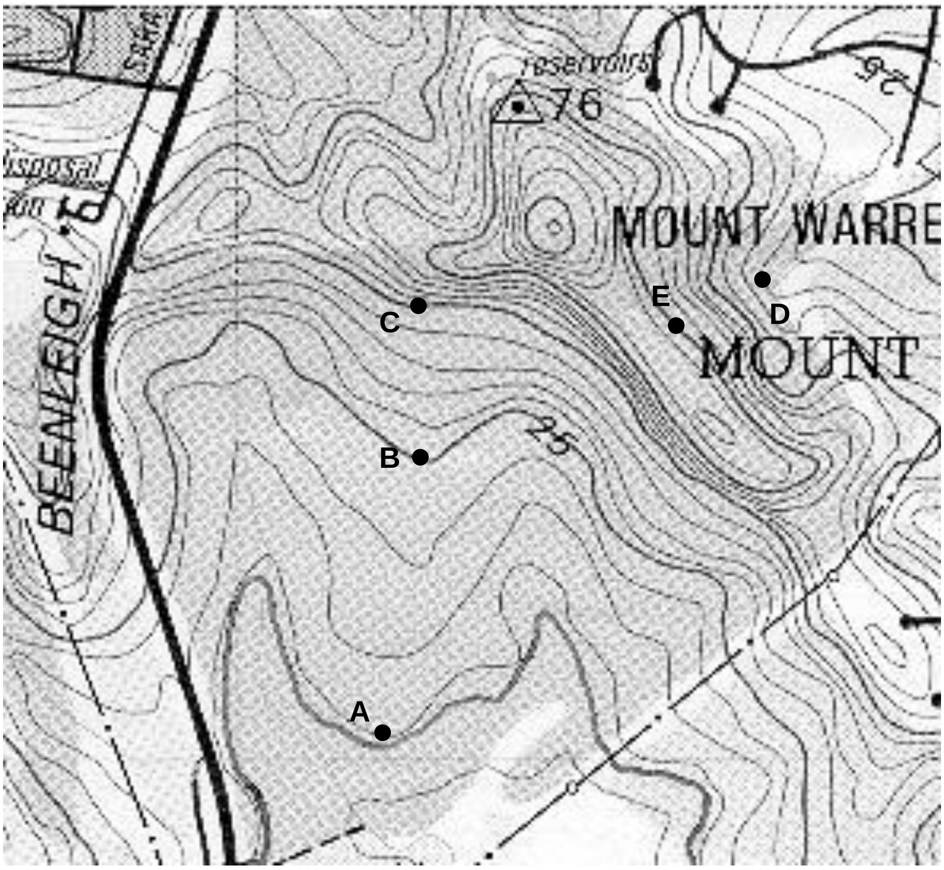
Another way of looking at this is that we just divide the total height by the total horizontal distance.

You will probably find that it was possible to walk up a plank this steep, so it would be possible to walk up the hill. You probably found that the maximum steepness for the plank that you could walk up was about 0.65 vertical metres per horizontal metre. So you could walk up the hill quite easily in fact, although if the ground is loose, that would make it harder, or if it is rocky and rough, that would make it easier, especially if you used your hands.

Exercise

Q1. Now it's your turn. There is a hill on the other side of Bahrs Scrub Road. You can just see the side of it on the bottom left corner of the map of the school area. The contours go up towards the corner of the map. The highest point near the corner of the map is about 125 m above sea level. It is not quite as steep as the hill behind the school. Work out the steepness of this hill.

Q2. The map below is of Mt Warren. The scale and contour intervals are the same as for the last map.



(a) Find the distance from A to B

(b) Find the difference in height between A and B

(c) Find the steepness of the ground between A and B

(d) Find the steepness of the ground between B and C

(e) Find the steepness of the ground between D and E

(f) Do closer contours mean that the ground is steeper or less steep?

(g) Where on the map is the ground steepest?

(h) How steep is it?

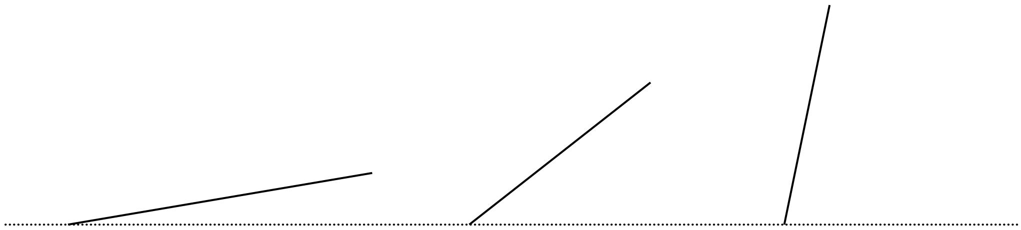
(i) Draw a bit of a topographic map at the same scale as the one above, but with the ground steepness 0.2 vertical metres per horizontal metre.

(j) What would a vertical cliff 50 m high look like on a topographic map?

Algebra – Lesson 5.4

Steepness of lines

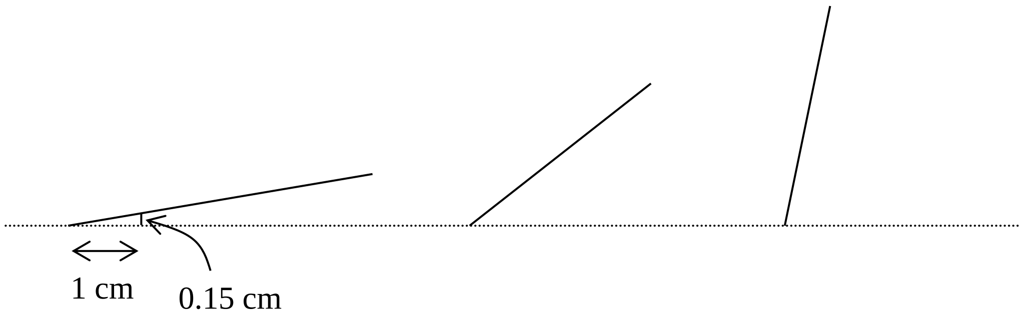
The lines below have different steepnesses. The one on the left is the least steep, the one on the right is the steepest .



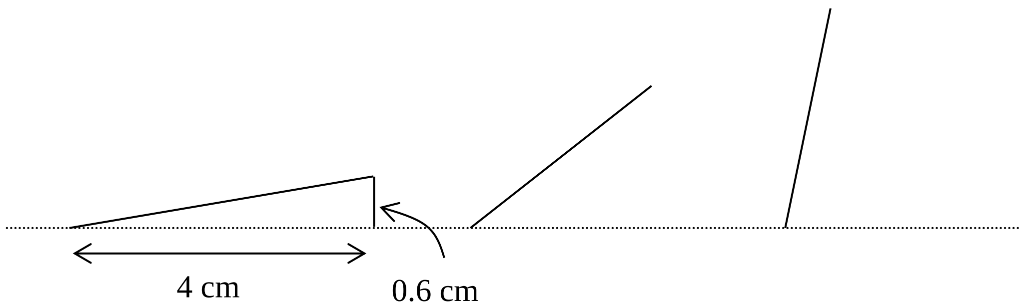
We can measure the steepness of lines the same way that we measure the steepness of slopes. It is the amount the line goes per horizontal metre.

If we wanted to measure the steepnesses of the lines above, we could try to measure 1 metre horizontally and see how many metres it has gone up. But of course the lines aren't long enough. We can get around that by using centimetres instead.

We can measure 1 cm horizontally on the first line, then measure how many centimetres it has gone up.



This gives a result of about 0.15 vertical centimetres per horizontal centimetre. Of course, using 1 cm is a bit fiddly and not very accurate. We could use the same technique as we used on the maps in the last lesson – measure the horizontal and vertical distances for the whole line.



Then we see that in moving 4 cm horizontally, we move 0.6 cm vertically. So in moving 1 cm horizontally, we must do a quarter of the distance, ie 0.15 cm. So the steepness still comes out to be 0.15 vertical centimetres per horizontal centimetre.
Once again what we have essentially done is divide the 0.6 by the 4.

Exercise

Q1. Find the steepness of the second line, firstly using just 1 cm, then using the whole length of the line and dividing. How close were your two results?

Q2. Find the steepnesses of the third line. Use the whole length of the line.

Q3. Now find the steepness of the three lines using millimetres. Again use the whole length of the lines – 1 mm would be much too small to see what you are doing!). Your steepnesses will be in vertical millimetres per horizontal millimetre.

Q4. Were you results from Q3 the same as the results when you used centimetres or were they different? Can you explain why?

Algebra – Lesson 5.5

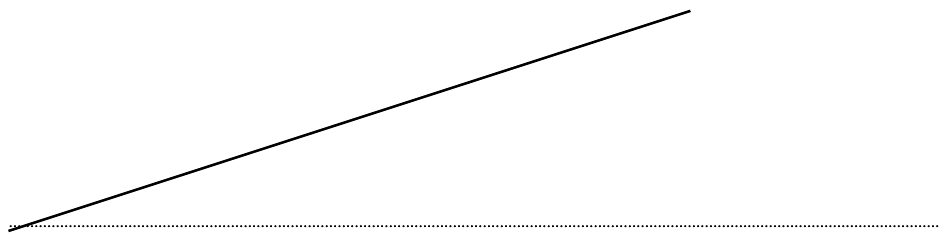
Gradient

In the last lesson, you should have realised that the units you use don't make any difference to the steepness of a line. If it goes up 0.5 cm in going horizontally 1 cm, then it will go up 0.5 m in going horizontally 1 m, and it will go up 0.5 km in going horizontally 1 km. In other words, however far it goes horizontally, it goes half as much vertically.

This means that we don't really need to say what units we are using when we say how steep a line is. We can just say the steepness in 0.5. We don't have to say 0.5 vertical metres per horizontal metre or whatever.

Exercise

Q1. Find the steepness of this line.



Give the answer as just a number.

Another way of measuring steepness

Of course, there is another way to say how steep the line above is. Instead of saying it has a steepness of .32 (or whatever you got for Q1), we could say that it makes an angle of 18° with the horizontal.

To distinguish these, we call the steepness the way we have been measuring it **the gradient** and we call the steepness measured as an angle **the slope angle**.

So in this unit we have been learning about gradients. Gradient just means steepness measured in vertical metres per horizontal metre (or vertical centimetres per horizontal centimetre or whatever).

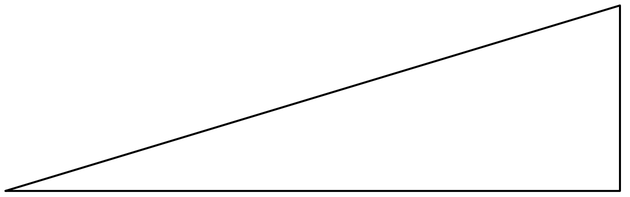
In fact we can define gradient as the number of units a line rises for each unit you move horizontally.

Gradient is
the number of units a line rises for each unit you move horizontally

You should already know this, but if you like definitions to remember, here it is.

You don't need to worry about slope angles until later.

Of course that is a definition of gradient, but a convenient way of calculating is to divide the distance risen vertically by the distance moved horizontally as in the following example:



Gradient = 25 ÷ 80 = 0.31

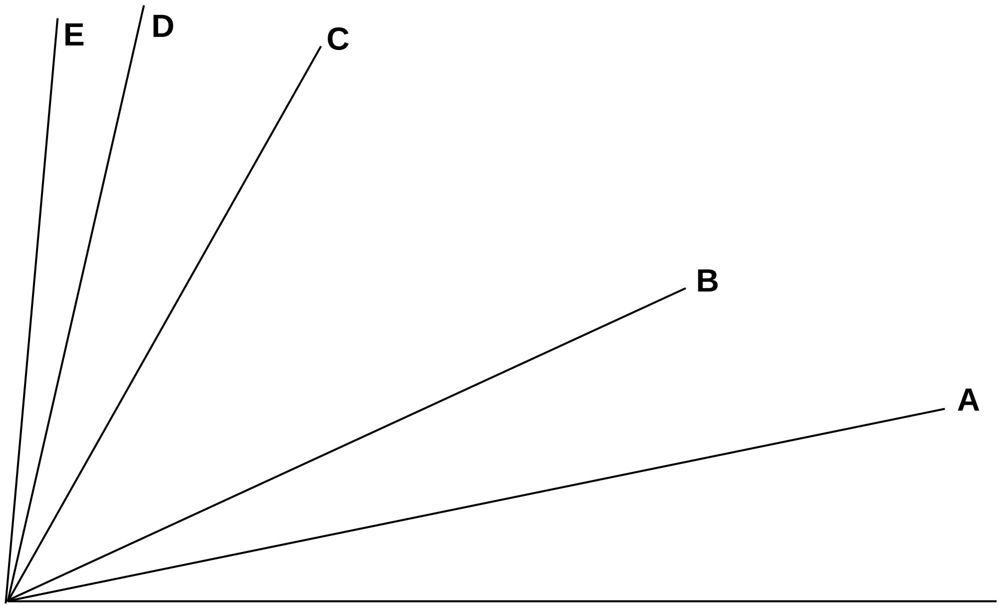
Sometimes the amount the line rises is called 'the rise' and the amount you move horizontally is called 'the run'. Then

$$gradient = \frac{rise}{run}$$

This is a nice easy way to remember how to find gradient. But don't forget its real meaning (in the grey box on the previous page).

Exercise

Q2. Find the gradients of these lines.



Q3. What is the gradient of a horizontal line?

Q4. What is the gradient of a vertical line?

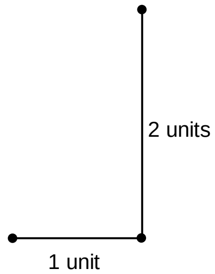
Q5. What is the gradient of a line that makes an angle of 45° with the horizontal?

Algebra – Lesson 5.6

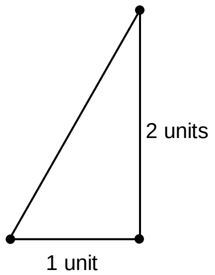
Drawing Lines with a given Gradient

So far, you have become an expert at finding the gradient of a line. But it is just as easy to draw a line with a specified gradient. For example, suppose we wanted to draw a line with a gradient of 2.

We mark one point as the start of our line. Then we measure 1 unit (any sort of unit – centimetre will do) to the right to a new point and two units up from the new point.

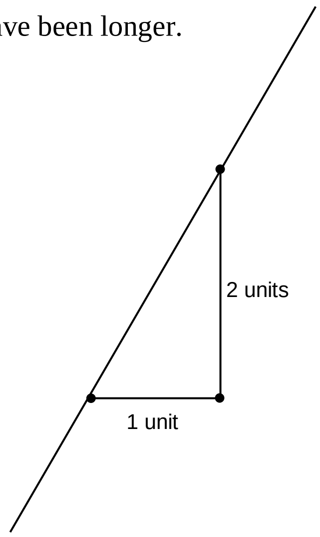


Then we join the first point to the last point.



Can you see why the new line has a gradient of 2?

Of course, the new line could have been longer.



Would it still have the same gradient?

Yes it would. Make sure you can understand why?

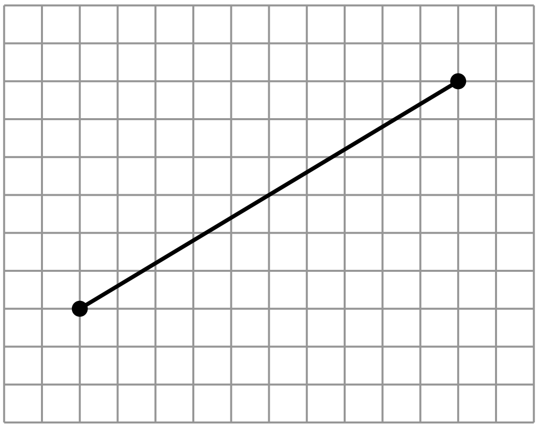
1 cm across and 2cm up makes a fairly small drawing. You can make it bigger by going say 5cm across and .

. .   How many centimetres up would you have to go to get a gradient of 2?

Exercise

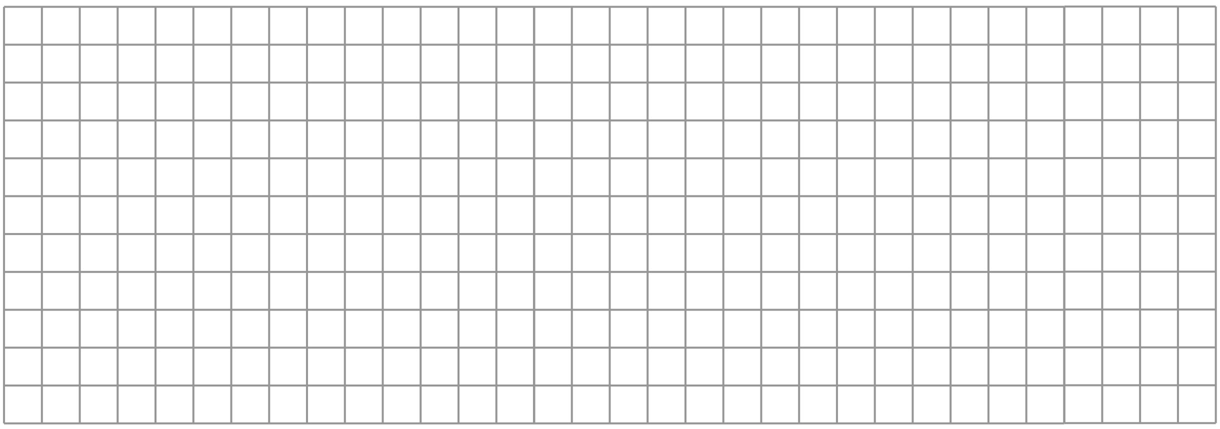
Q1. Draw lines with gradients of
(a) 2 (b) 3 (c) 0.5

It is easier to draw lines of a given gradient on a grid. You don't have to measure then – just count squares. For example, to draw a line of gradient 0.6 on the grid below, pick a point where the lines cross, then count one square across and 0.6 squares up. Or to get it more accurate, count 10 squares across and 6 up.



Exercise

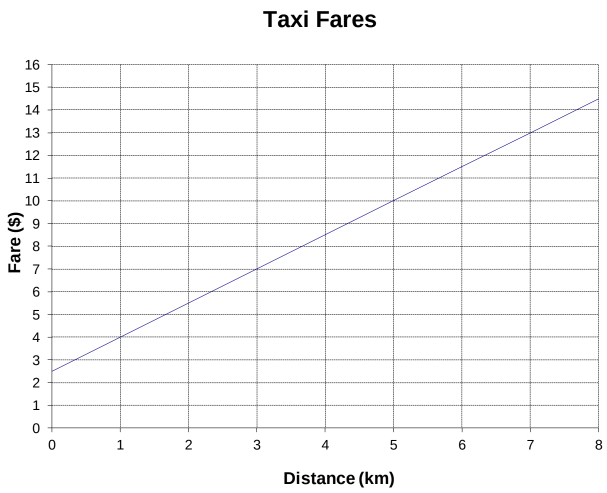
Q2. On the grid below draw lines with the following gradients:
(a) 1 (b) 0.4 (c) 1.5 (d) 4 (e) 0 (f) 10
Label each one.



Algebra – Lesson 5.7

Formulae from Graphs

Many of the relations we have looked at have produced straight lines when we graph them. This taxi fare graph from Unit 1 is an example



Can we express this relation as a formula?

Let's have a look. If you don't go any distance, the fare is $2.50. We can see this by reading off the fare when the distance is zero. This is the flag fall – what the metre is set to when you get in the taxi.

How much extra are you charged if you go 1 km. Well, looking at the graph, we see that the fare for 1 km is $4.00. This is $1.50 more than the flag fall, so the metre clicks over $1.50 in going 1 km. We can see (and we would expect) that the metre will click over another $1.50 for thew next kilometre, and another $1.50 for the next and so on. In other words, you are charged $2.50 plus $1.50 multiplied by the number of kilometres. If we let the distance be d and the fare be f, we have

$$f = 2.5 + d \times 1.5$$

This is the formula. Make sure this makes sense before you go on – it is crucial!

What we did here was read off the value for f when $d = 0$. This gives the first number in the formula.
Then we found out how much the line rises as we move one unit horizontally. This gives the second number in the formula.

The first number is where the line cuts the vertical axis. This is called the vertical axis intercept.
You might have noticed that the second number is the gradient of the line.

So the formula can be written down by finding the vertical axis intercept and the gradient and writing:

$$f = \text{vertical axis intercept} + d \times \text{gradient}$$

In fact every relation which has a straight line graph has a formula like this:

$$\textit{dependent variable} = \text{vertical axis intercept} + \textit{independent variable} \times \text{gradient}$$

Exercise

Q1. Find the formulae for the following relations

5.7 graph → formula
5.8 formula → graph (quick method)
5.9 table → formula; formula → table

5.10 x and y

Algebra – Lesson 5.8

Graphs from Formulae

You already know how to convert a formula to a graph. You pick a few values of the independent variable and work out the corresponding values of the dependent variable, then plot the values on the graph and, if it is continuous, join them up.

Exercise

Just to refresh your memory, draw graphs of the following relations

Q1. $f = 5 + 3d$ where f is the fare and d the distance travelled

Q2. $A = 3.14r^2$ where A is the area of a circle and r is its radius

Assume that both relations are continuous.

But there is a much quicker way to graph the first relation. It uses what you learnt in the last lesson about the vertical axis intercept and the gradient.

dependent variable = vertical axis intercept + *independent variable* $\times$ gradient

The number by itself, 5, is the vertical axis intercept and the number multiplied by the independent variable is the gradient.

So we know that the vertical axis intercept is 5 and the gradient is 2. What we do is mark the point 5 units up the vertical axis. This is one point on the graph. To get another point, move one unit to the right and 3 units up (because the gradient is 3) and mark that point. You can then move another unit to the right and another 3 units up and mark another point and then again and again, but the points will all be on the same straight line, so you could just draw a long line through the first two points.

Exercise

Use this method to draw a graph of $f = 2d + 4$ for values of d from 0 to 5