Real and Complex Numbers – Diagnostic Test

Q1. Put ticks in the appropriate boxes. Leave the others blank.

	–6	0.3877	$\sqrt{5}$	2π	$\sqrt{-3}$	$\sqrt{16}$	∞
Natural number							
Whole number							
Integer							
Rational number							
Irrational number							
Surd							
Real number							
Imaginary number							
Complex number							

Q2. Prove that $\sqrt{11}$ is irrational.

Q3. Simplify

(a) $\sqrt{80}$

(b) $\dfrac{2+\sqrt{3}}{4-\sqrt{3}}$

Q4. Simplify

(a) i^{51}

(b) $(3 + 2i) - (2 - i)$

(c) $(3 + i)^2$

Q5. Prove that $\overline{w}\,\overline{z} = \overline{wz}$ where $w,z \in \mathbb{C}$

Q6. Write $(2 - 2i)^8 \times (1 + i)$ in trigonometric form.

Q7. Solve $\dfrac{z+1}{z-i} = 2$

Q8. Write $\sqrt{i}$ in Cartesian form

Solutions

Q1. Put ticks in the appropriate boxes. Leave the others blank.

	–6	0.3877	$\sqrt{5}$	2π	$\sqrt{-3}$	$\sqrt{16}$	∞
Natural number						✓	
Whole number						✓	
Integer	✓					✓	
Rational number	✓	✓				✓	
Irrational number			✓	✓			
Surd			✓				
Real number	✓	✓	✓	✓		✓	
Imaginary number					✓		
Complex number	✓	✓	✓	✓	✓	✓	

Q2. Prove that $\sqrt{11}$ is irrational.

Assume $\sqrt{11}$ is rational, ie. it can be written as a/b where a and b are integers with no common factors

$\frac{a}{b} = \sqrt{11}$

$a = \sqrt{11}.b$

$a^2 = 11b^2$

$\therefore$ a^2 has a factor of 11, and so a has a factor of 11

So we can write a as 11k where k is an integer

$a^2 = (11k)^2$

$= 11^2k^2$

$a^2 = 11b^2$

$\therefore 11b^2 = 11^2k^2$

$b^2 = 11k^2$

$\therefore$ b has a factor of 11

This contradicts the original assumption and shows that $\sqrt{11}$ is not rational.

Q3. Simplify

(a) $\sqrt{80}$

$= \sqrt{16}\sqrt{5}$

$= 4\sqrt{5}$

(b) $\frac{2+\sqrt{3}}{4-\sqrt{3}}$

$= \frac{2+\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}$

$= \frac{8+4\sqrt{3}+2\sqrt{3}+3}{13}$

$= \frac{11+6\sqrt{3}}{13}$

$= \frac{11}{13} + \frac{6\sqrt{3}}{13}$ (optional step)

Q4. Simplify

(a) i^{51}

$= i^{48} i^{3}$

$= i^{3}$

$= -i$

(b) $(3 + 2i) - (2 - i)$

$= 1+3i$

(c) $(3 + i)^2$

$= (3+i)(3+i)$

$= 9+6i-1$

$= 8+6i$

Q5. Prove that $\overline{wz} = \overline{w}\,\overline{z}$ where $w,z \in \mathbb{C}$

Let $w = a+bi$, $z = c+di$

$\overline{w} = a-bi$, $\overline{z} = c-di$

$LHS = \overline{w}\,\overline{z} = (a-bi)(c-di)$

$= ac-bd-(ad+bc)i$

$RHS = \overline{wz} = \overline{(a+bi)(c+di)}$

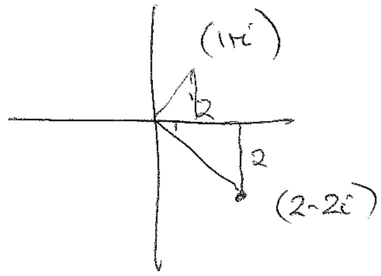
$= \overline{ac-bd+(bc+ad)i}$

$= ac-bd-(ad+bc)i$

$= LHS$

QED

Q6. Write $(2-2i)^8 \times (1+i)$ in trigonometric form.



$2-2i = 2\sqrt{2}\,\text{cis}(-\pi/4)$

$1+i = \sqrt{2}\,\text{cis}\,\pi/4$

$(2-2i)^8 = (2\sqrt{2})^8\,\text{cis}\,0$

$(2-2i)^8(1+i) = (2\sqrt{2})^8\,\text{cis}\,0 \times \sqrt{2}\,\text{cis}\,\pi/4$

$= 4096\sqrt{2}\,\text{cis}\,\pi/4$

Q7. Solve $\dfrac{z+1}{z-i} = 2$

Let z be a+bi

$a+bi+1 = 2(a+bi-i)$

$a+1+bi = 2a+2(b-1)i$

$a+1 = 2a$

$\therefore a = 1$

$2(b-1) = b$

$\therefore 2b-2 = b$

$b = 2$

$\therefore z = a+2i$

Q8. Write $\sqrt{i}$ in Cartesian form

Let $\sqrt{i}$ be $a+bi$

$(a+bi)^2 = i$

$(a+bi)(a+bi) = i$

$a^2-b^2+2abi = i$

$a^2-b^2 = 0$ $\quad 2ab = 1$

$b = 1/2a$

$a^2 - \frac{1}{4a^2} = 0$

$a^2 = \frac{1}{4a^2}$

$a^4 = 1/4$

$a^2 = 1/2$

$a = 1/\sqrt{2}$

$2/\sqrt{2}\,b = 1$

$b = \frac{2}{\sqrt{2}}$

$\therefore \sqrt{i} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$