1. (a) Calculate $3 \text{ cis } \frac{\pi}{2} \times \frac{1}{2} \text{ cis } \frac{\pi}{6}$, giving the answer in trigonometric form.

   (b) Express $(1 - \sqrt{3}\, i)^{12}$ in Cartesian form.

2. Solve $z^2 = 2z - 2$

3. Solve $z^4 = 2\sqrt{2} + 2\sqrt{2}\, i$, giving the solutions in trigonometric form.

4. Draw the graph of $|z - 2 + i| = 2$.

5. Write $\sin x + \sin y$ as a product of trigonometric functions. Hence find the exact value of $\sin 75° + \sin 15°$. Give the answer in simplest form.

6. Use trigonometric identities to find the exact value of $\tan\left({}^{5\pi}/_{12}\right)$. Give the answer in simplest form.

7. Prove that $\cot A + \cot B = \dfrac{\sin(A+B)}{\sin A \sin B}$.

8. Prove by induction that $1 + 2 + 3 + 4 + \ldots + n = \frac{n}{2}(n + 1)$.

9. Prove by induction that, $\forall n \in N$, $n(n^2 + 2)$ is a multiple of 3.

10. Jethro gets a job modelling underwear. He starts on a salary of \$34 900 per year. At the end of each year's service, he gets a 4.2% pay rise. If he keeps the job for 16 years,
    (a) how much will he earn in his 12^{th} year in the job?
    (b) how much will he earn altogether over the 16 years?

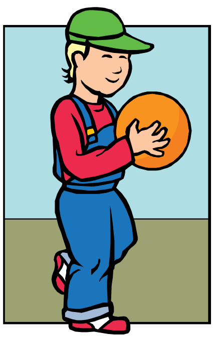
1. June is Albert's exercise month. Last year he decided to walk. He did a modest walk on June 1, then, each day after that, he increased the distance by a fixed amount. By the end of June 10, he had walked a total of 77 km. By the end of June 20, he had walked a total of 214 km. His wife, May, knew a bit of maths and decided to work out how far he would have walked by the end of the month. May said that an arithmetic sequence was involved. She said that between 10^{th} and 20^{th}, Albert's total had increased by 137 km, and therefore it would increase by the same amount between 20^{th} and 30^{th}. Therefore, he would have walked 351 km by the end of the month.
Was May's argument valid? If not, what was wrong with it and what should her answer have been?

2. Harriet thought of a number, cubed it, then added 6 times the original number. This gave her the square of her original number. What was her original number?

3. Find $\int \sin x \sin 3x \cos 6x \, dx$

4. Prove that $\frac{2}{\sqrt{4-4\cos^2\theta}} = \frac{1+\cos 2\theta}{2\sin\theta} + \sin\theta$.

5. A ball is dropped from a height of 1 m. On its first five bounces it is measured to rise to 80 cm, 64 cm, 51 cm, 41 cm, 33 cm.
   (a) Develop a model for the height of the nth bounce. Give reasons for your choice.
   What assumptions were necessary in developing your model?
   What are the strengths and limitations of the model?
   (b) According to your model
      (i) How many times will it bounce?
      (ii) How far will it travel in total?



Extra questions

KAPS

1. Jethro gets a job modelling underwear. He starts on a salary of \$34 900 per year. At the end of each year's service, he gets a 4.2% pay rise. If he keeps the job for 26 years,
   (a) how much will he earn in his 18^{th} year in the job?
   (b) how much will he earn altogether over the 26 years?

2. Prove by induction that $1 + 2 + 3 + 4 + \ldots + n = \frac{n}{2}(n+1)$.

3. Prove by induction that, $\forall n \in N$, $n\,(n^2 + 2)$ is a multiple of 3 .

4. Write the expansion of $\sin x + \sin y$. Hence find the exact value of $\sin 75° + \sin 15°$. Give the answer in simplest form.

5. Use trigonometric identities for find the exact value of $\tan\left(\,{}^{5\pi}/_{12}\right)$. Give the answer in simplest form.

6. Prove that $\cot A + \cot B = \dfrac{\sin(A+B)}{\sin A \sin B}$.

7. Express $(1 - \sqrt{2}\,i)^{12}$ in Cartesian form.

8. Solve $z^4 = 4 + 4i$, giving the solutions in exact trigonometric form.

9. Draw the graph of $|z - 2 + i\,| < 2$.

MAPS

10. The sum of the first 10 terms of an arithmetic sequence is 80; the sum of the first 15 terms is 45. Find the sum of the first 20 terms. [$a = 17$, $d = -2$, Ans = –40]

11. Prove that $\operatorname{cosec}\theta - \sin\theta = \dfrac{1+\cos 2\theta}{2\sin\theta}$.

12. Find $\int \sin x \sin 3x \cos 6x \, dx$

13. Harriet thought of a number, cubed it, then added 6 times the original number. This gave her the square of her original number. What was her original number?

14. A ball is dropped from a height of 1 m. Each time it bounces, it rises to ${}^4/_5$ of the previous height.
    (a) How many times will it bounce?
    (b) How far will it travel in total?
    (c) Will it ever stop bouncing? If so, how long will it take? If not, how high will it be bouncing
    one minute after being dropped? [assume $g = 9.8$ m/s^2]

Ex 6A Q5(a-f),6-8.

15. The 10^{th} term of an AP is 41 and the sum of the first 6 terms is 363. What is the 60^{th} term? [a=68, d=-3, ans=-109]

16. The sum of the first 12 terms of an AP is 2 and the sum of the first 100 terms is 2000. What is the 50^{th} term?

17. Fat Harry goes on an exercise program, but gets lazy. He walks 10 km the first day. Each subsequent day he walks 10% less than the previous day.
    (a) How far will he walk on the 20^{th} day?
    (b) How far will he walk in total in the first 20 days?
    (c) Assuming he keeps this up for ever, how far will he walk altogether?

18. A swinging door is opened 90°. When let go, it swings back 150° (so it is 60° from the closed position), then it swings forward 120° (so it is 60° from the closed position), then back 96° (so it is 36° from the closed position) and so on, each swing being $^4/_5$ of the previous swing. How far from the closed position does it stop? [6.666°]

19. Bronson leaves the pub and walks 1 km east, ½ km south, 250 m west, 125 m north, 62.5 m east and so on in a spiral pattern. How far from the pub does he end up?

12 Maths C Term 1 Paper A 2012

Q1 a $3\operatorname{cis}\frac{\pi}{2} \times \frac{1}{2}\operatorname{cis}\frac{\pi}{6}$

$= 3\times\frac{1}{2}\operatorname{cis}\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$

$= \frac{3}{2}\operatorname{cis}\frac{2\pi}{3}$

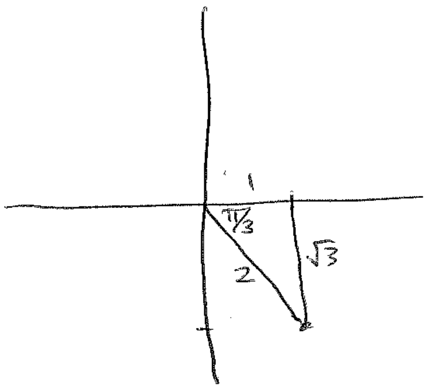
b $1-\sqrt{3}i$

$= 2\operatorname{cis}\left(-\pi/3\right)$

$2\operatorname{cis}\left(-\pi/3\right)^{12}$

$= 4096\operatorname{cis}\left(-4\pi\right)$

$= 4096$



Q2

$$z^2 = 2z - 2$$

$$z^2 - 2z + 2 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{\cancel{4}\,2 \pm \sqrt{4 - 8}}{2}$$

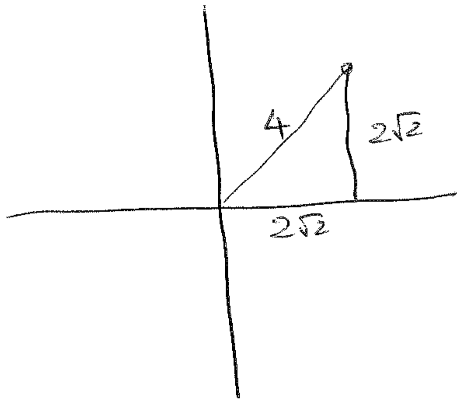
$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

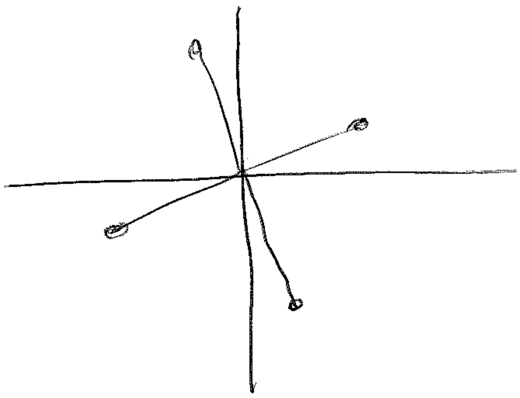
$$= 1 \pm i$$

Q3 $z^4 = 2\sqrt{2} + 2\sqrt{2}i$

$= 4 \operatorname{cis} \pi/4$



$z^4 = \sqrt{2} \operatorname{cis} \pi/16$



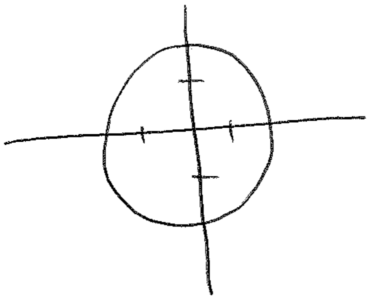
$z^4 = \sqrt{2} \operatorname{cis} \pi/16$

or $\sqrt{2} \operatorname{cis} 9\pi/16$

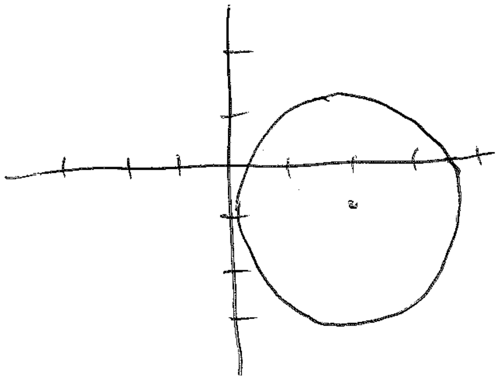
or $\sqrt{2} \operatorname{cis} 17\pi/16$

or $\sqrt{2} \operatorname{cis} 25\pi/16$

Q4 $|z-2+i|=2$



$|z| = 2$



$|z-2+i| = 2$

Q5

$$\sin x + \sin y = 2\sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y)$$

$$\sin 75^\circ + \sin 15^\circ = 2\sin 45^\circ \cos 30^\circ$$

$$= 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{6}}{2}$$

Q6

$$\tan\frac{5\pi}{12} = \tan\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$$

$$= \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \frac{\tan(\pi/4) + \tan \pi/6)}{1 - \tan(\pi/4)\tan(\pi/6)}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{3+2\sqrt{3}+1}{2}$$

$$= 2+\sqrt{3}$$

Q7 RTP $\cot A + \cot B = \dfrac{\sin(A+B)}{\sin A \sin B}$

$$\text{RHS} = \frac{\sin A \cos B + \cos A \sin B}{\sin A \sin B}$$

$$= \frac{\sin A \cos B}{\sin A \sin B} + \frac{\cos A \sin B}{\sin A \sin B}$$

$$= \frac{\cos B}{\sin B} + \frac{\cos A}{\sin A}$$

$$= \cot B + \cot A$$

$$= \cot A + \cot B$$

$$= \text{LHS}$$

QED

Q8 RTP $1+2+3+\ldots+n = \frac{n}{2}(n+1)$

RTP it is true for $n=1$

ie $1 = \frac{1}{2}(1+1)$

This is true

RTP that if it is true for $n=k$, then it is true for $n=k+1$

Assume it is true for $n=k$

ie $1+2+3+\ldots+k = \frac{k}{2}(k+1)$

RTP it is true for $n=k+1$

ie $1+2+3+\ldots+k+k+1 = \left(\frac{k+1}{2}\right)(k+1+1)$

$$\text{LHS} = \frac{k}{2}(k+1)+k+1$$

$$= \frac{k^2}{2}+\frac{k}{2}+k+1$$

$$= \frac{k^2}{2}+\frac{3k}{2}+1$$

$$\text{RHS} = \frac{1}{2}(k+1)(k+2)$$

$$= \frac{1}{2}(k^2+3k+2)$$

$$= \frac{k^2}{2}+\frac{3k}{2}+1$$

$$= \text{LHS} \qquad \text{QED}$$

$\therefore$ It is true for all n.

Q9 RTP $n(n^2+2)$ is a multiple of 3

RTP it is true for $n=1$

ie $1(1^2+2)$ is a multiple of 3

$1\times(1^2+2) = 3$

So it is true for $n=1$

RTP ~~that~~ that if it is true for $n=k$, then it is true for $n=k+1$

Assume it is true for $n=k$

ie $k(k^2+2)$ is a multiple of 3

$(k+1)((k+1)^2+2)$

$= k(k^2+2k+1+2) + (k^2+2k+1+2)$

$= k^3 + 2k^2 + 3k + k^2 + 2k + 3$

$= k^3 + 3k^2 + 3k + 3 + 2k$

$= k^3 + 2k + 3(k^2 + k + 1)$

$= k(k^2+2) + 3(k^2+k+1)$

The first term is ~~divisible by~~ a multiple of 3 (assumed)

The second term is ~~clearly divisi~~ a multiple of 3

So it is true for $n=k+1$

$\therefore$ It is true for all $n \in \mathbb{N}$

Q10 His annual salary is a GP

$a = 34900 \quad r = 1.042$

In his 12^{th} year he will earn

$a \times r^{12-1}$

$= 34900 \times 1.042^{11}$

$= \$54\,874.47$

His total earnings over the 16 years will be

$$\frac{a(r^{16}-1)}{r-1}$$

$$= \frac{34\,900 \times (1.042^{16}-1)}{0.042}$$

$= \$773\,990.72$

12 Maths C Term 1 Paper B 2012

Q1 Her argument is not valid because she is assuming that the sums of an arithmetic sequence form an arithmetic sequence. This is not the case.

Let the distance walked on 1 June be a

Let the daily increase be d

$$S_{10} = \frac{10}{2}[2a+9d] = 77 \quad \ldots \quad (1)$$

$$S_{20} = \frac{20}{2}[2a+19d] = 214 \quad \ldots \quad (2)$$

$(1) \Rightarrow \quad 10a + 45d = 77$

$(2) \Rightarrow \quad 20a + 190d = 214$

Solving gives $a = 5 \quad d = 0.6$

$$S_{30} = \frac{30}{2}[2a+29d]$$

$$= 15(10+17.4)$$

$$= 411$$

So by the end of the month, he would have walked 411 km

Q2

Let her number be z

$$z^3 + 6z = z^2$$

$$z^3 - z^2 + 6z = 0$$

$$z(z^2 - \cancel{3}z + 6) = 0$$

$$z = 0 \text{ or } z^2 - z + 6 = 0$$

$$a = 1 \quad b = -1 \quad c = 6$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 - 24}}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\sqrt{23}\,i \text{ or } \frac{1}{2} - \frac{1}{2}\sqrt{23}\,i$$

$$\therefore z = 0 \text{ or } \frac{1}{2} + \frac{1}{2}\sqrt{23}\,i \text{ or } \frac{1}{2} - \frac{1}{2}\sqrt{23}\,i$$

Q3

$$\sin x \sin 3x = \tfrac{1}{2}\cos 2x - \tfrac{1}{2}\cos 4x$$

$$\sin x \sin 3x \cos 6x = \left(\tfrac{1}{2}\cos 2x - \tfrac{1}{2}\cos 4x\right)\cos 6x$$

$$= \tfrac{1}{2}\left[\cos 2x \cos 6x - \cos 4x \cos 6x\right]$$

$$= \tfrac{1}{2}\left[\tfrac{1}{2}\cos 8x + \tfrac{1}{2}\cos 4x - \tfrac{1}{2}\cos 10x - \tfrac{1}{2}\cos 2x\right]$$

$$= \tfrac{1}{4}\left[\cos 4x + \cos 8x - \cos 2x - \cos 10x\right]$$

$$\int \sin x \sin 3x \cos 6x \, dx$$

$$= \tfrac{1}{4}\int (\cos 4x + \cos 8x - \cos 2x - \cos 10x)\, dx$$

$$= \tfrac{1}{4}\left[\tfrac{1}{4}\sin 4x + \tfrac{1}{8}\sin 8x - \tfrac{1}{2}\sin 2x - \tfrac{1}{10}\sin 10x\right] + c$$

$$= \tfrac{1}{16}\sin 4x + \tfrac{1}{32}\sin 8x - \tfrac{1}{8}\sin 2x - \tfrac{1}{40}\sin 10x + c$$

Q4 RTP $\dfrac{2}{\sqrt{4-4\cos^2\theta}} = \dfrac{1+\cos 2\theta}{2\sin\theta} + \sin\theta$

$$\text{LHS} = \frac{2}{2\sqrt{1-\cos^2\theta}}$$

$$= \frac{1}{\sin\theta}$$

$$\text{RHS} = \frac{1+\cos 2\theta + 2\sin^2\theta}{2\sin\theta}$$

$$= \frac{1+(1-2\sin^2\theta)+2\sin^2\theta}{2\sin\theta}$$

$$= \frac{2}{2\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \text{LHS}$$

QED

Q5

(a) The heights of the bounces approximate a GP with $a = 80$ $r = 0.8$. This would give

bounce	1	2	3	4	5
height	80	64	51.2	40.96	32.768

This choice fits the data closely and might be expected from the situation.

It needs to be assumed that the measured heights were approximations measured to the nearest Centimetre. It is also assumed that the pattern will continue

~~A~~ Strengths of the model are that.

- it is simple to use
- it fits the data closely
- it can be extended to subsequent bounces.

A limitation is that it doesn't give exactly the measured heights

(b) The model is a GP. Accordingly, it will have an infinite number of terms. So according to the model, it will bounce an infinite number of times.

The total distance is the sum of ~~the~~ a GP plus the initial drop

$a = 1.6 \quad r = 0.8$

$$S_\infty = \frac{a}{1-r} = \frac{1.6}{0.2} = 8$$

$$\text{Total distance travelled} = 8 + 1$$
$$= 9\text{m}.$$